# Statistics

# Lecture 12

Feb 19-8:47 AM

SG 21

16% of 250 randomly Selected Students were STEM majors. $n = 250$ $\hat{p} = .16$

$x = n\hat{p} = 250(.16) = 40$

If decimal → Round-up

C-level: .9

Find 90% Conf. interval for the prop. of all Students that are STEM majors.

STAT TESTS 1-PropZInt

$x = 40$

$n = 250$

C-level: .9

Calculate

$.12 < p < .20$

$E = \frac{.20 - .12}{2} = .04$

$\hat{p} = \frac{.20 + .12}{2} = .16$

Since $\hat{p}$ was in 2-dec, we round to 2-dec.

We are 90% Confident that between 12% & 20% of all Students are STEM majors.

Nov 18-6:51 PM

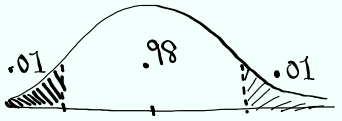
Find min. Sample Size ($n$) needed if we wish to construct 98% Conf. interval for the Prop. of all Students that are STEM majors and error not to exceed 5%.

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.16)(.84)\left(\frac{2.326}{.05}\right)^2$$

if decimal → Round-up

$= 290.856\ldots$

$n \approx 291$




$z_{.01} = \text{invNorm}(.99, 0, 1) = 2.326$

Suppose $\hat{p}$ & $\hat{q}$ are both unknown, $\hat{p} = .5$, $\hat{q} = .5$

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{2.326}{.05}\right)^2 = 541.0276$$

$n \approx 542$

SG 21 ✓

Nov 18-7:01 PM

SG 22

Estimating Population Mean $\mu$:

Answer ... $< \mu <$ ...

Format $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample Mean, Point-estimate

$E$: Margin of error

**Case I: $\sigma$ Known**

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

CV $z_{\alpha/2}$ → invNorm

Conf. Interval → Z Interval

input: STATS

$$E = \frac{-}{2}$$

$$\bar{x} = \frac{+}{2}$$

Nov 18-7:11 PM

Given $n=32$ $\bar{x}=84$ $\sigma=12$

C-level: .99

Find 99% conf. interval for pop. mean $\mu$.

$\_\_ < \mu < \_\_$

Since $\sigma$ is known,

STAT TESTS ZInterval

inpt: Stats

$\sigma=12$

$\bar{x}=84$ (whole)

$n=32$

C-level: .99

Calculate

$78.536 < \mu < 89.464$

→ Round to whole #

$79 < \mu < 89$

$E = \frac{89-79}{2} = 5$

$\bar{x} = \frac{89+79}{2} = 84$

Nov 18-7:17 PM

A Sample of 35 Students had a mean age of 31.8 yrs.

$n=35$, $\bar{x}=31.8$

No C-level: .95

Find Conf. interval for the mean age of all Students if standard deviation of ages of all Students is 7.5 yrs.

$\sigma=7.5$

$\_\_ < \mu < \_\_$

$\sigma$ is known → ZInterval

inpt: Stats

$\sigma=7.5$

$\bar{x}=31.8$ (1-dec.)

$n=35$

C-level: .95

Calculate

→ Round to 1-dec.

$29.3 < \mu < 34.3$

$E = \frac{34.3-29.3}{2} = 2.5$

$\bar{x} = \frac{34.3+29.3}{2} = 31.8$

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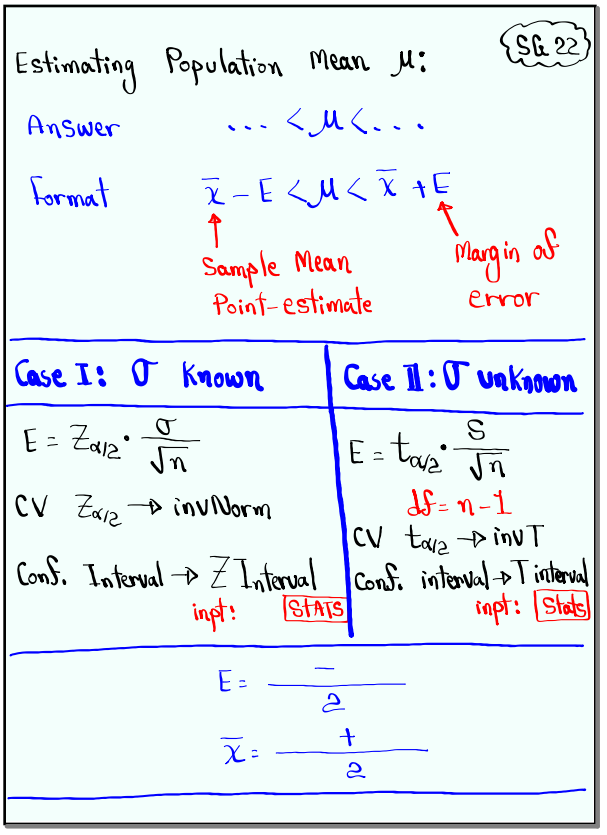


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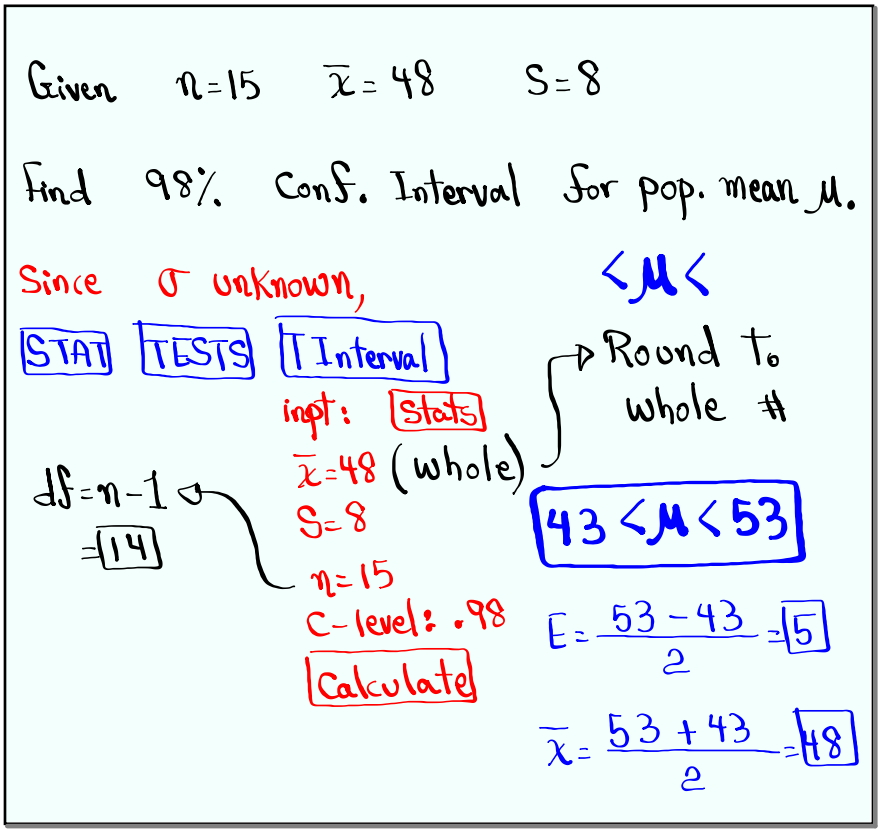



Nov 18-7:39 PM

I randomly selected 12 nurses, their mean age was 48.5 yrs with standard dev. of 8.5 yrs.

$n = 12$ $\bar{x} = 48.5$ $S = 8.5$ (1-dec.)

No C-level → .95

Find Conf. interval for the mean age of all nurses.

$\sigma$ unknown → T Interval

$df = n - 1 = 11$

$43.1 < \mu < 53.9$

$E = \frac{53.9 - 43.1}{2} = 5.4$

$\bar{x} = \frac{53.9 + 43.1}{2} = 48.5$

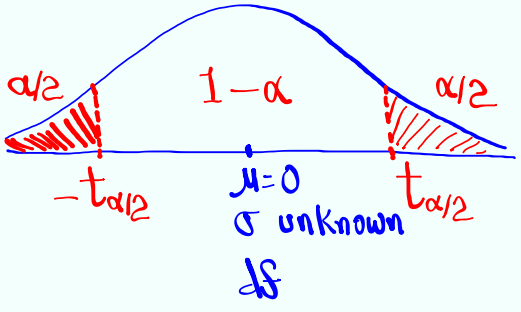
Nov 18-7:46 PM

t-Dist.:

1) Symmetric, bell-shape, Total Area = 1

2) $\mu = 0$, $\sigma$ unknown

3) It comes with degrees of freedom. (df)




To find $t_{\alpha/2}$

2nd VARS

invT(Left Area, df)

Nov 18-7:55 PM

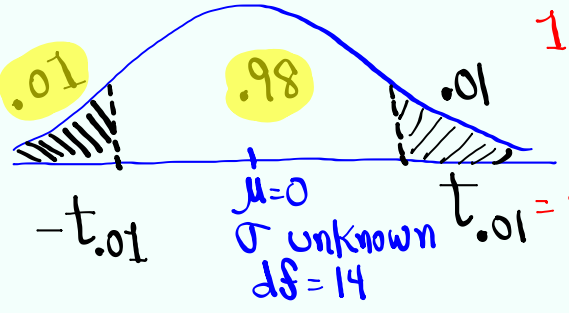
Find $t_{.01}$ with $df = 14$.

→ Right-Tail Area

→ $\alpha/2 = .01$ → $\alpha = .02$ Sig. level

$1 - \alpha = .98$ Middle Area

$.98 = 98\%$ C-level




$t_{.01} = \text{invT}(.99, 14)$

$= \boxed{2.624}$

Nov 18-8:00 PM

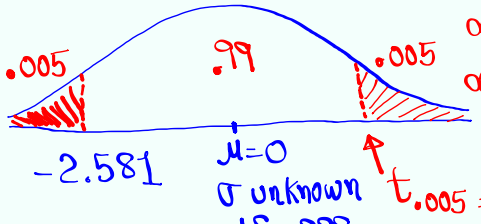
Find $\pm t_{\alpha/2}$ for 99% C-level and $df = 999$.

Middle Area = .99

$1 - \alpha = .99$

$\alpha = .01$ Sig. level

$\alpha/2 = .005$ area of each tail.




$t_{.005} = \text{invT}(.995, 999)$

$= \boxed{2.581}$

As df increases,

$t_{\alpha/2} \approx Z_{\alpha/2}$

Nov 18-8:05 PM

## what is degrees of freedom?

In Statistics, it is the number of values that are free to Vary.

It can be computed by certain formulas Per topic or it is given.

ex: 20 students, I bring 20 donuts.

| | |
|---|---|
| First student | 20 choices |
| Second " | 19 choices |
| Third " | 18 " |
| ⋮ | ⋮ |
| Last student | 0 choice (1 donut left) |

df = 19

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7 clean shirts, You wear one clean shirt Per day.

| | |
|---|---|
| Monday | 7 choices |
| Tuesday | 6 " |
| wednesday | 5 " |
| ⋮ | ⋮ |
| Sunday | 0 choices (1 clean shirt) |

df = 6

Nov 18-8:29 PM

How to determine min. Sample Size for Constructing Conf. interval for Pop. mean $\mu$:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

with Some algebra

$$n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

if decimal → Roond-up

If $\sigma$ is unknown, use $s$ in place of $\sigma$

$$n = \left( \frac{z_{\alpha/2} \cdot s}{E} \right)^2$$

Nov 18-8:32 PM

Given $\sigma = 20$, $E = 2.5$

Find Sample Size $n$ to construct 90% Conf. interval for pop. mean $\mu$.

$$n = \left( \frac{z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left( \frac{1.645 \cdot 20}{2.5} \right)^2 = 173.1856 \approx \boxed{174}$$


$z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

Redo with $E = 5$.

$$n = \left( \frac{1.645 \cdot 20}{5} \right)^2 = 43.2964 \quad \boxed{n \approx 44}$$

Nov 18-8:36 PM

Given $S = 18$ $E = 10$

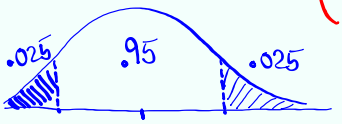
Find min. Sample Size needed to Construct Conf. interval for Pop. mean $\mu$.

No C-level → .95

$\sigma$ unknown $\quad n = \left(\frac{z_{\alpha/2} \cdot S}{E}\right)^2$

$= \left(\frac{1.960 \cdot 18}{10}\right)^2 = 12.44\ldots$

$n = 13$




$z_{.025} = \text{invNorm}(.975, 0, 1)$

$= 1.960$

SG 22

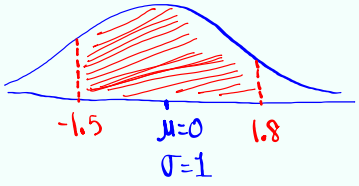
Redo with $E = 5$

$n = \left(\frac{1.960 \cdot 18}{5}\right)^2 = 49.7\ldots$ $\quad n = 50$

Nov 18-8:45 PM

Find $P(-1.5 < Z < 1.8)$

$= \text{normalcdf}(-1.5, 1.8, 0, 1)$




$= .897$

Find $P(-1.5 < t < 1.8)$ with $df = 19$.

2nd VARS


$\text{tcdf}(-1.5, 1.8, 19)$

Lower, upper, df

$= .881$

Redo with $df = 999$

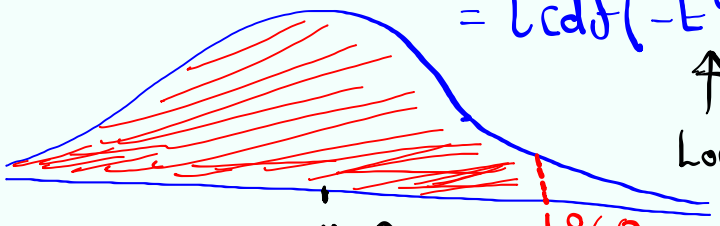
$\text{tcdf}(-1.5, 1.8, 999) = .897$

Nov 18-8:55 PM

Find $P(t < 1.960)$ with $df = 49$.

$= tcdf(-E99, 1.960, 49)$

Lower upper df



$\mu = 0$
$\sigma$ unknown
$df = 49$

1.960

$= .972$

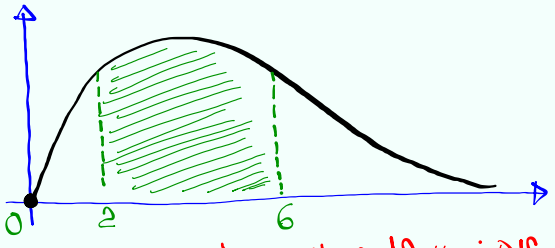
Nov 18-9:04 PM

F - Dist.:

1) Starts at 0, Skewed to right.

2) Not Symmetric, total Area = 1

3) It comes with two deg. of freedom
Ndf Ddf




Find $P(2 < F < 6)$ with Ndf = 4 & Ddf = 20

$= fcdf(2, 6, 4, 20)$
L U Ndf Ddf

2nd VARS

$= .131$

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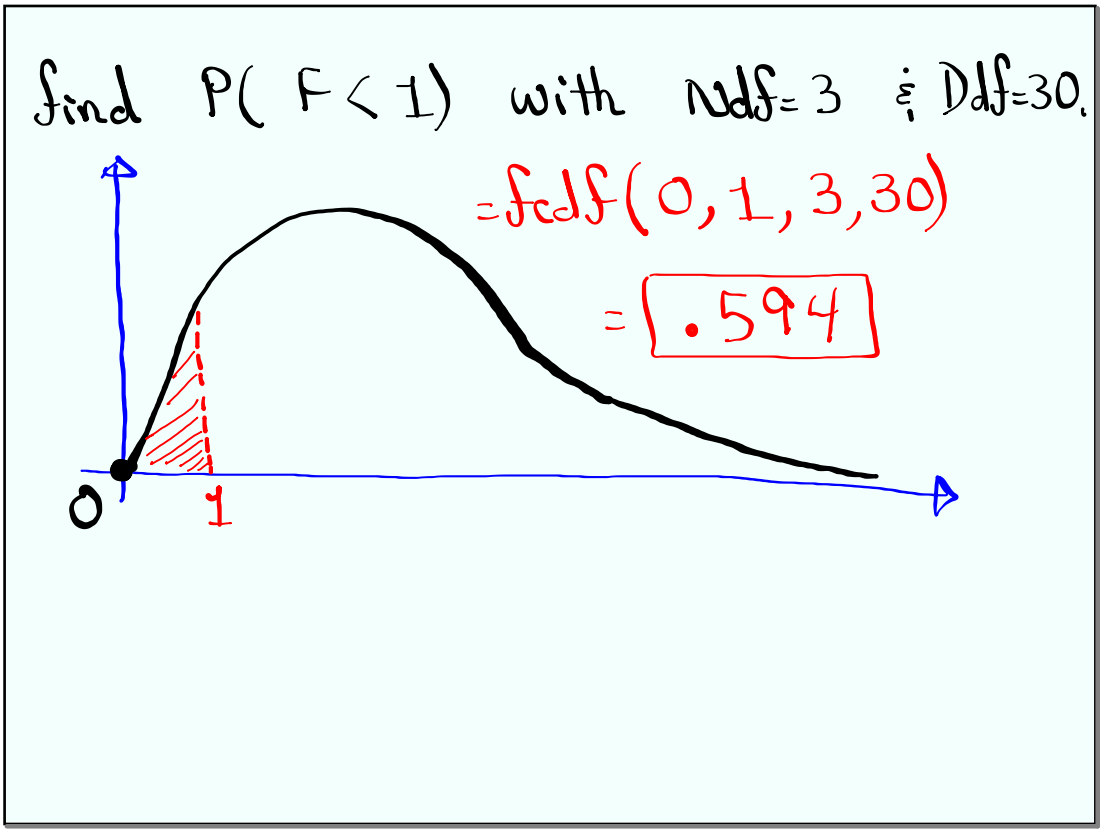


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